SOME FEATURES OF ATMOSPHERICS WAVEFORMS TRANSFORMATION IN THE PROCESS OF THEIR PROPAGATION OVER THE EARTH AND ITS INFLUENCE ON THE ACCURACY OF DTOA LIGHTNING LOCATION SYSTEMS

I.I. Kononov, V.I. Ivanov, D.M. Krutoy and I.E. Yusupov
Saint-Petersburg State University, Research Institute of Radiophysics
Saint-Petersburg, Russia

The report is devoted to the analysis of lightning electromagnetic pulse (LEMP) transformation in a process of its propagation in the Earth-ionosphere waveguide at distances up to ten thousands kilometers and its possible influence on the accuracy of lightning multi-station DTOA systems. We will consider the main features of this transformation in relation to such important parameters of DTOA location systems as detection efficiency and location accuracy focusing more attention to their far-zone (trans-continental) versions.

1. ACCURACY CHARACTERISTICS OF DTOA LOCATION SYSTEMS

As it is well known the estimation of lightning stroke location in a set of \( N \) spaced observation sites of any type multi-station system can be found as a solution of the set of equations:

\[
T_i(x) = T_i, \quad (1.1)
\]

where \( T_i \) is the value (supposed to be constant in the given expression) of some measured parameter, \( i = 1, \ldots, N_i \). \( N_i = N - 1 \) for direction finding (DF) systems and \( N_i = N - 1 \) for difference time of arrival (DTOA) ones. Every equation in (1.1) defines in two-dimensional variant a position line, and in three-dimensional one — a surface. Hereinafter we will consider (taking into account far-distant location) two-dimensional variants only, acceptable for the majority of practical applications.

Supposing that measurements in all registration sites are accidental and independent, matrix of measurements errors is the diagonal one:

\[
C_T = \begin{pmatrix}
\sigma^2_{T_1} & 0 & \ldots & 0 \\
0 & \sigma^2_{T_2} & \ldots & 0 \\
\ldots & \ldots & \ldots & \ldots \\
0 & \ldots & \ldots & \sigma^2_{T_N}
\end{pmatrix}. \quad (1.2)
\]

Here \( \sigma_{T_i}^2 \) — is the dispersion of measured parameter \( T_i \).

A covariation matrix \( C_x \) of evaluated coordinates of radiation source vector \( \vec{x} \) is related with covariation matrix \( C_T \) of measured parameters \( T_i \) by the expression:

\[
C_x = [A^T C_T^{-1} A]^{-1}, \quad (1.3)
\]

where terms \( a_{ik} = \frac{dT_i}{dx_k} \) of matrix \( A \) are calculated in the point, for which location error is defined. Taking into account the above mentioned suggestions one can transform (1.3) to the form

\[
C_x = \left( \sum_i \sum_k \frac{\sin^2(\phi_i - \phi_k)}{\sigma_{q_k}^2} \right)^{-1} \times \\
\left( \sum_i \frac{\sin^2 \phi_i}{\sigma_{q_i}^2} + \sum_k \frac{\sin \phi_k \cos \phi_i}{\sigma_{q_k}^2} \right) \left( \sum_i \frac{\sin \phi_k \cos \phi_i}{\sigma_{q_i}^2} + \sum_k \frac{\cos^2 \phi_i}{\sigma_{q_k}^2} \right), \quad (1.4)
\]

here \( \phi_i \) — is an angle of tangent to \( i \)-th position line in the considered point;
\( \sigma_{q_i}^2 \) — is an accidental displacement of this position line. Its value is related to \( \sigma_{T_i} \) by the expression

\[
\sigma_{q_i}^2 = \sigma_{T_i}^2 \left| \text{grad} T_i \right|, \quad (1.5)
\]

For DTOA systems the measured parameter \( T_{kl} \) is the difference of distances between radiation source and \( k, l \) observation sites

\[
T_{kl} = R_k - R_l; \quad \left| \text{grad} T_{kl} \right| = 2\sin \left( \frac{\eta_{kl}}{2} \right), \quad (1.6)
\]

where \( \eta_{kl} \) — is the angle, under which a system’s base (the line between observation sites) is seen from the radiation source position.

Supposing that velocity of registered radio signals propagation is constant and equal to the speed of light \( c \) the measured value \( T_{kl} \) may be submitted as:

\[
T_{kl} = c \cdot \left( t_k - t_l \right), \quad (1.7)
\]
In this case the position lines are hyperbolas.

For rough comparative evaluations of system’s errors the matrix (1.4) can be replaced by its trace, herewith an error’s ellipse is replaced by a circle of radius $\sigma$, defined by the expression:

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} \sum_{i'}^{n} \sin^2 (\phi_i - \phi_{i'}) \sigma_{ii'}^2.$$  (1.8)

This expression can be used for estimation of DTOA system’s accuracy, which includes caused by propagation effects.

The determination in different registration sites of systematic errors of signal’s time of arrival research is an estimation of possible sources characteristics. The main objective of this sites, keeping the same accuracy increasing an operative zone of location advantages of DTOA systems. It allows transformation of similarity. It is one of the picture of systems inaccuracy is kept at their centers. One can note that specified proportionally to a square of distance from area of both systems the errors increase including all registration sites. In the external (approximately within the limits of the circle realized in the internal area of DTOA system the least location errors (less than 1 km) are given in Fig.1. As it is seen from this picture, for some hypothetic 3- and 4-sites systems are supposed to be equal $\sigma_3$:

$$\sigma_3^2 = \left( \frac{\sin^2 \eta_{12} + \sin^2 \eta_{13} + \sin^2 \eta_{23}}{2} \right)^2 \left( \frac{\sin^2 \eta_{12} + \sin^2 \eta_{13} + \sin^2 \eta_{23}}{2} \right)^2 A,$$  (1.9)

where

$$A = \sin^2 \eta_{12} \sin^2 \eta_{13} \sin^2 (\phi_{12} - \phi_{23}) + \sin^2 \eta_{12} \sin^2 \eta_{13} \sin^2 (\phi_{12} - \phi_{13}) + \sin^2 \eta_{23} \sin^2 \eta_{13} \sin^2 (\phi_{23} - \phi_{13}).$$

The schematic plane pictures of location errors calculated (in suggestion that $\sigma_i = 1\mu$s) for some hypothetic 3- and 4-sites systems are given in Fig.1. As it is seen from this picture, the least location errors (less than 1 km) are realized in the internal area of DTOA system (approximately within the limits of the circle including all registration sites). In the external area of both systems the errors increase proportionally to a square of distance from their centers. One can note that specified picture of systems inaccuracy is kept at transformation of similarity. It is one of the advantages of DTOA systems. It allows increasing an operative zone of location system by increasing base distances between sites, keeping the same accuracy characteristics. The main objective of this research is an estimation of possible sources of systematic errors of signal’s time of arrival determination in different registration sites caused by propagation effects.

2. PROPAGATION OF VLF SIGNALS.
THEORETICAL BASE

The radiation source is supposed to be the point vertical electric dipole located over the Earth’s surface and described as

$$p_x = P_x (2\pi r^2 \sin \theta)^{-\frac{1}{2}} \delta (r - a) \delta (\theta - 0).$$

The model of a spherical waveguide with a radially inhomogeneous anisotropic ionosphere (but regular along propagation path) has been used for numerical calculations. The spherical cavity of the Earth-ionosphere waveguide between values $r = a, d$ is considered to have the properties of a free space. The Earth ($r < a$) is considered to be homogeneous and is characterized by a conductivity $\sigma$, the ionosphere ($r > d$) is considered to be a radially inhomogeneous anisotropic media characterized by the complex permittivity tensor $\hat{\varepsilon}$, elements of which are assumed to be conjugated if the geomagnetic field vector $H_0$ changes its sign (in dependence on the direction of electromagnetic wave propagation).

In the frame of this waveguide model and impedance boundary conditions at $r = a$, and at some ionospheric boundary at $r = d$ given as

$$\begin{pmatrix} E_{\theta} \\ E_{\varphi} \end{pmatrix}_{r=a} = \hat{\delta} \begin{pmatrix} \hat{H}_{\theta} \\ \hat{H}_{\varphi} \end{pmatrix}, \begin{pmatrix} \hat{H}_{\theta} \\ \hat{H}_{\varphi} \end{pmatrix}_{r=d} = \hat{\delta} \begin{pmatrix} E_{\theta} \\ E_{\varphi} \end{pmatrix},$$  (2.1)

were the dimensionless impedance $\hat{\delta}$ and admittance $\hat{\delta}$ are 2 by 2 matrices, the radial (vertical) electric field component $E_r$ considered in the report is described as an expansion in series (a superposition of TM and TE modes):

$$E_r = \frac{P_x}{4\pi k \beta r^2} \sum_{n=0}^{\infty} R_n^{(c)} (k R) R_n^{(c)} (k R) \times \left[ n a_1 R_n^{(c)} (x) + R_n^{(c)} (x) \right] \times \left[ R_n^{(c)} (x) + n a_1 R_n^{(c)} (x) \right] + a_1 a_2 R_n^{(c)} (x) R_n^{(c)} (x) = 0$$  (2.2)

for $x = x_2 = \frac{a}{\beta}$. Using asymptotic forms of the Legendre functions valid for $|x| \theta >> 1$, $|x| (\pi \theta) >> 1$:
\[
P_\nu(\cos \theta) = \sqrt{2}(\pi \nu \sin \theta)^{1/2} \cos(\nu + \sqrt{2} \theta - \pi/4),
\]
(2.4)
neglecting the round-the-world and backward waves and taking into account that \( |\nu| \gg ka \), the radial component (2.2) becomes:
\[
E_r = A k^2 P \sum_{\nu=1} \Lambda_\nu f'^{\nu}(kr) e^{i\nu \theta},
\]
(2.5)
where
\[
A = \frac{1}{2\pi b r^2 \sin \theta} \int \psi r dr,
\]
\[
\psi_r = \left( \nu + \frac{1}{2} \right) \theta - kR,
\]
\( R = a\theta \).

Quantities \( \Lambda_\nu \) and \( f'^{\nu}(x) \) in the expression (2.5) represent the excitation factors and height-gain functions, respectively, and can be expressed by the radial functions \( R_\nu^{(\nu)}(x) \):
\[
\Lambda_\nu = \frac{2\pi a S_{\nu}^{3/2}}{2k N_\nu} e^{i\nu \theta},
\]
\[
\tilde{N}_\nu = \frac{ka^2 N_\nu}{\left[ R_\nu^{(\nu)}(ka) \right]^2},
\]
\( f'^{\nu}(x) = R_\nu^{(\nu)}(x)[R_\nu^{(\nu)}(ka)]^{-1} \).

We will not discuss here the fine details of the calculation algorithm, note only that it is based on the methods and analytical expressions given in Galuk and Ivanov 1978; Rybachek et al. (1997).

3. RESULTS OF NUMERICAL CALCULATIONS

The model numerical calculations of an electric field radial (vertical) component \( E_r \) were done by expression (2.5) using a function \( P_\nu(t) = P_\nu(at)^2 e^{i\nu \theta} \) as approximation of a source's dipole moment. Duration of the first halfwave of this function's second derivative (practically identical to the waveform of a signal \( E_r \) registered at distances 50...100 km) is in the ratio with parameter \( a \) as \( t_0 = 1.26/a \). Therefore variations of this parameter at numerical calculations within the limits of 2·10^4...10^5 sec^{-1}, correspond to possible variations of types of real atmospherics in the enough wide range. Taking value of \( P_\nu = 0.5 \cdot 10^9 a^2 \) (C·km) one can normalize amplitude of the first halfwave of signal's waveform at 100 km to 1 V/m (for the perfectly conducting ground). Besides for convenience we will present all calculated waveforms for any distance \( R \) (km) as values \( E_r \) normalized to 100 km (\( E_r^* = E_r / R/100 \)).

Here we give some calculation data at the distance range 100...9000 km for the midday (zenith angle \( \chi = 14^\circ \)) and midnight (\( \chi = 127^\circ \)) propagation conditions. The number of members in series expansion (2.5) used for calculations was 11 for day and 30 for night conditions. The curves characterizing electron density profiles \( N_\nu(h) \) for different zenith angles (taken from Rawer et al (1978)) are given in Fig.2. The values of the Earth conductivity were taken as \( \sigma = 4 \) S/m for sea propagation paths and \( 10^{-2}, 10^{-3} \) S/m for the land ones.

![Fig.2. Ionosphere electron density profiles](image)

Several examples of atmospherics waveforms calculated for the distance ranges 100...900 km and 1000...9000 km for the day and night propagation conditions and the ground conductivity \( \sigma = 10^{-2} \) S/m are given in Figs.3,4 (\( a = 3 \cdot 10^4 \) sec^{-1}).

One can observe a significant dispersion of signals propagating at great distances. They become wider, a quality of single halfwaves increases with distance, and the serial number of halfwave of the greatest amplitude increases too. Fig.5 illustrates such a kind dependence on the distance of serial number \( N \) of the maximal signal’s halfwave. Dependencies on distance of absolute values of maximal amplitudes of calculated signals are resulted in Fig.6.

As it follows from calculations an attenuating of propagating signals is more significant in the day conditions rather than in the night ones. For example, at 5000 km the ratio of maximal amplitudes of signals propagating over ground with \( \sigma = 10^{-2} \) S/m in the night and day conditions is about 1.7; at 9000 km this value increases to 3.3. Conductivity of the Earth also affects rather
significantly (see Fig.6). So, the ratio of amplitudes of signals propagating over sea \((\sigma = 4 \text{ S/m})\) and land \((\sigma = 10^{-3} \text{ S/m})\) is about 4.2 at night and increases to 10 at day conditions. These features are more pronounced in pictures of amplitudes of normalized \(E_r^*\) signals (see Fig.7), in which the inverse dependence \(1/R\) from distance is excluded.

4. DISCUSSION AND CONCLUSION

First it is necessary to note that for DTOA systems with distances between observation sites no more than 1000 km it is preferable to use for a signal's time of arrival determination the characteristic points formed in limits of the first (ground) halfwave. For the arrival time determination one can use the time position of the signal's front \(t_f\), (determined with using of the secant going through the levels 0.1 and 0.9 of the ground wave front), the time position of a ground wave maximum \(t_m\), or the position of the first (after maximum) zero-crossing \(t_0\). In the worst case it can be the moment of crossing a signal of some fixed threshold level. As it follows from the estimations given in Kononov and Petrenko (1992), there is some additional delay of these points (against a position of the point propagating with velocity of light) depending on the ground conductivity. These delays depend on the ground conductivity. They make for \(t_f - 0.25\) and \(0.6 \mu\text{sec}\) on every 100 km increments of range accordingly for \(\sigma = 10^{-2}\) and \(10^{-3} \text{ S/m}\). Corresponding values for \(t_m\) are 0.37 and \(0.81 \mu\text{sec}\) and for \(t_0 - 1.2\) and \(1.5 \mu\text{sec}\).

For larger distances between registration sites and to the radiation source the problem of arrival time determination becomes more complicated. The ground wave strongly decays, especially for ground conductivities smaller than \(10^{-3} \text{ S/m}\) and from distances of 4...5 thousand kilometers practically disappears, being less than level of external noise. In this case a time position of absolute signal's maximum or following zero-crossing can be used for arrival time determination. However in case of a possible great difference of distances from a source to registration sites or the wrong identification of characteristic points their time position can be determined with rather great error reaching several tens of microseconds, which can lead to the great mistakes of lightning location. We will not give here concrete values of these errors. They can be easily estimated by means of dependences (via distance) of zero-crossings of separate half waves given in Fig.8. This kind of location error can be excluded after its rough preliminary estimation by means of special algorithm.

Once more important problem related with TDOA system is increasing of its detection efficiency. For its decision it is necessary to have information on laws of change of atmospherics amplitudes with distance. As it follows from calculations (and illustrations given in Fig.6) it is possible to allocate three intervals of distances in which character of change amplitude with range noticeably varies. The first interval is in the range up to 300...400 km, where one can use \(1/R\) approximation. The value of \(k\) depends on the type of signal and of conductivity of ground. For waveforms corresponding to return strokes and conductivities from \(4\) to \(10^{-2} \text{ S/m}\) value of \(k\) is near to 1. For more high-frequency signals (leaders and k-pulses) and conductivities about \(10^{-3} \text{ S/m}\) its value can achieve 1.4. The second interval is in the range 400...1000 km. The amplitude attenuation in this interval is not so strong and can be roughly described by two exponential approximation. The third interval 2000...10000 km can be approximated by the function \((1/R)*\exp(-aR)\), where \(a = 10^{-4} \text{ km}^{-1}\) for the day propagation conditions and \(0.6·10^{-4} \text{ km}^{-1}\) – for the night ones (for ground conductivity \(\sigma = 10^{-2} \text{ S/m}\)).

REFERENCES


Fig. 1. Zones of equal accuracy of coordinates estimation (for 1 µsec error of arrival time determination). Color gradation corresponds to error changing by 1 km.

Fig. 3. Waveforms of normalized electric field component $E^*$ calculated for the night propagation conditions at distance intervals 100...900 km (left picture) and 1000...9000 km (right picture).

Fig. 4. Waveforms of normalized electric field component $E^*$ calculated for the day propagation conditions at distance intervals 100...900 km (left picture) and 1000...9000 km (right picture).
Fig. 5. Dependence on distance of serial number $N$ of the maximal halfwave of signals for midnight (left picture) and midday (right picture) propagation conditions for different conductivities $\sigma = 4 \text{ S/m}$ (dashed line), $\sigma = 10^{-2} \text{ S/m}$ (solid line), $\sigma = 10^{-3} \text{ S/m}$ (dotted line).

Fig. 6. Dependence of absolute value of maximal amplitude of signal via distance calculated for midday and midnight conditions for different values of conductivity (1 — $\sigma = 4 \text{ S/m}$, 2 — $\sigma = 10^{-2} \text{ S/m}$, 3 — $\sigma = 10^{-3} \text{ S/m}$, 4 — $\sigma = 10^{-5} \text{ S/m}$).

Fig. 7. Dependence of absolute value of maximal amplitude of normalized signal via distance calculated for $\sigma = 10^{-2} \text{ S/m}$ for different values of a source approximation parameter $a$ equal to: $a_1 = 2 \cdot 10^4$, $a_2 = 3 \cdot 10^4$, $a_3 = 10^5 \text{ sec}^{-1}$.

Fig. 8. Dependence on distance of time positions of second (1), third (2) and fourth (3) zero-crossings counted from the beginning of a signal, propagating with a speed of light for the values $\sigma = 4 \text{ S/m}$ (dashed line), $\sigma = 10^{-2} \text{ S/m}$ (solid line), $\sigma = 10^{-3} \text{ S/m}$ (dotted line) and parameter $a = 3 \cdot 10^4 \text{ sec}^{-1}$.